Maths GCSE (Higher) Learning Resource

University of Northampton

Basic Surd Rules

$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

$\sqrt{a \div b} = \sqrt{a} \div \sqrt{b}$

$\sqrt{a} \times \sqrt{a} = a$

Rationalising Surds

$\frac{a}{\sqrt{b}}$ Get rid of the surd on the bottom by multiplying top and bottom by that surd.

$\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

This resource was developed in conjunction with the maths subject teaching staff at Sir Christopher Hatton Academy 2017-18

National Collaborative Outreach Programme

Rationalising Surds

$\frac{a}{\sqrt{b}+c}$ Get rid of the surd expression on the bottom by multiplying by that surd expression but changing the sign.

$\frac{a}{\sqrt{b}+c} \times \frac{\sqrt{b}-c}{\sqrt{b}-c} = \frac{a\sqrt{b}-ac}{b-c}$

Basic Index Laws

$a^m \times a^n = a^{m+n}$ $\quad a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$ $\quad a^{-m} = \frac{1}{a^m}$

$a^0 = 1$

Fractional Index Laws

$a^{\frac{1}{2}} = \sqrt{a}$ $\quad a^{\frac{1}{3}} = \sqrt[3]{a}$ $\quad a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ $\quad a^{\frac{m}{n}} = \frac{1}{(\sqrt[n]{a})^m}$

Compound Interest

final amount = starting amount x % multiplier $^{\text{number of years}}$

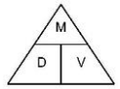
Example

£5000 is invested in a savings account. It earns compound interest at a rate of 4% per annum. How much will be in the account after 6 years?

£5000 x 1.04^6 = **£6326.60**

Density, Mass and Volume

density = mass ÷ volume



density – g/cm³, kg/m³

mass – grams (g), kilograms (kg)

volume – cm³, m³

DON'T MARRY VAMPIRES!

Direct & Inverse Proportion

y is proportional to x

- $y \propto x$
- $y = kx$

Substitute values to find k.

y is inversely proportional to x

- $y \propto \frac{1}{x}$
- $y = \frac{k}{x}$

Substitute values to find k.

Key Word Facts

ALGEBRA	YOU NEED TO KNOW
Expression	Numbers and letters, has NO = sign
Equation	Numbers and letters HAS AN = sign
Substitute	To replace a letter with a number
Like Terms	Collect the same combination of letters together and all the numbers together
Simplify	Collect all the like terms
Expand	Get rid of the brackets (multiply out brackets)
Factorise	Put Brackets back in

Simultaneous Equations

Different
Add
Same
Subtract

Expanding Double Brackets

$(a + 4)(a + 2)$

$= a^2 + 2a + 4a + 8$

$= a^2 + 6a + 8$

Factorising Quadratics with a Coefficient

$6x^2 + 17x + 12$

72

1	72
2	36
3	24
4	18
6	12
8	9

$6x^2 + 8x + 9x + 12$

$(6x^2 + 8x) + (9x + 12)$

$2x(3x + 4) + 3(3x + 4)$

$(3x + 4)(2x + 3)$

Steps:

1) Find factors of 72 that add up to 17

2) Rewrite the polynomial so that the middle term is a sum of the 2 factors you found

3) Factor by grouping

Quadratic Formula

A quadratic equation is in the form

$ax^2 + bx + c = 0$

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the Square

Complete the square: $x^2 + 6x - 11 = 0$

- Half the coefficient of x and put in brackets

$(x + 3)^2 - 11$

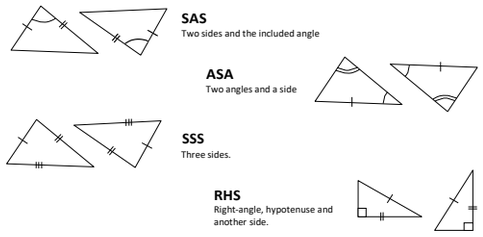
- Square the number in the bracket and subtract it

$(x + 3)^2 - 11 - 3^2$

$(x + 3)^2 - 11 - 9$

$(x + 3)^2 - 20$ The turning point of this graph will be (-3, -20)

Congruency Rules



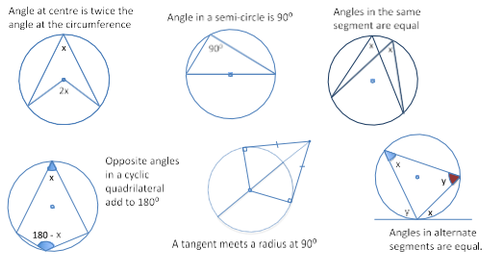
SAS
Two sides and the included angle

ASA
Two angles and a side

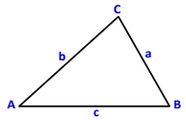
SSS
Three sides.

RHS
Right-angle, hypotenuse and another side.

Circle Theorems



Sine Rule

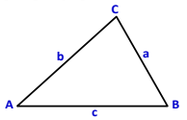


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

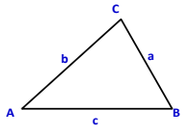
Cosine Rule



Side $!^2 = \$^2 + \&^2 - 2\$\,\&)*+$

Angle $\cos A = \frac{1^2 3 4^2 5 6^2}{^{\prime\prime}14}$

Area of a Triangle



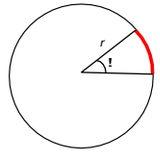
Area $= \frac{1}{2}\$\%\&'()$

Standard Trigonometric Values

	0°	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-

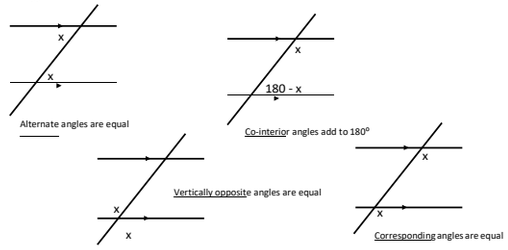
Sectors of Circles

$$\text{Area} = \pi r^4 \times \frac{\theta}{360}$$



$$\text{Arc length} = 2\pi r \times \frac{\theta}{\cancel{360}}$$

Angles in Parallel Lines



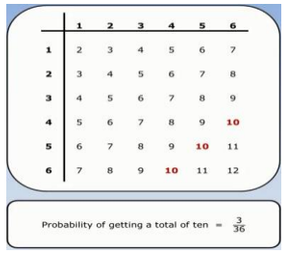
Probability Rules

the 'and' rule: P(A) **and** P(B) = P(A) x P(B)

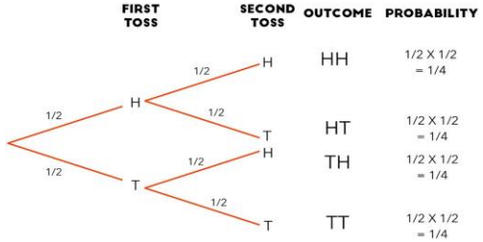
the 'or' rule: P(A) **or** P(B) = P(A) + P(B)

Sample Space Diagram

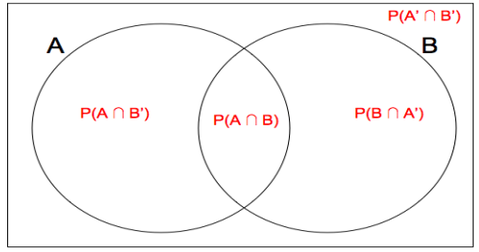
shows all outcomes



Tree Diagrams



Venn Diagram Notation



Averages from a Grouped Frequency Table

Weight, w, Kg	Frequency
$40 < w \leq 50$	2
$50 < w \leq 60$	15
$60 < w \leq 70$	18
$70 < w \leq 80$	10
$80 < w \leq 90$	2

Modal Class
Group with the highest frequency
$60 < w \leq 70$

Class in which the Median lies
$(\frac{1^{\prime\prime}\#}{\$})$ where n is the number of values
$(\frac{\%\&^{\prime\prime}\#}{\$}) = 24^{th}$ value
Count up from frequency
24^{th} value is in $60 < w \leq 70$

Averages from a Grouped Frequency Table

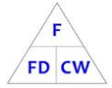
Weight, w, Kg	Frequency	
$40 < w \leq 50$	45 2	90
$50 < w \leq 60$	55 15	825
$60 < w \leq 70$	65 18	1170
$70 < w \leq 80$	75 10	750
$80 < w \leq 90$	85 2	170
	47	**3005**

Estimate for the Mean
- Find midpoints of groups
- Multiply midpoint by frequency
- Divide total of (midpoint x frequency) by total frequency

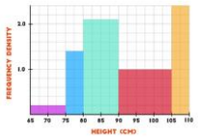
3005 ÷ 47 = 63.9kg (1 dp)

Histograms Frequency density

$= \frac{!^{\prime\prime}\#\$\%\#\&'(}{^{\prime})*++\ ,-\text{-}/0}$

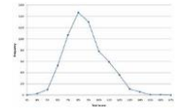


The area of each bar will give the frequency!



Graphs from Grouped Data

Frequency polygon → plot midpoints (x) with frequency (y), join with straight lines



Cumulative frequency → plot upper bound of each group (x) with cumulative frequency (y), join with curve