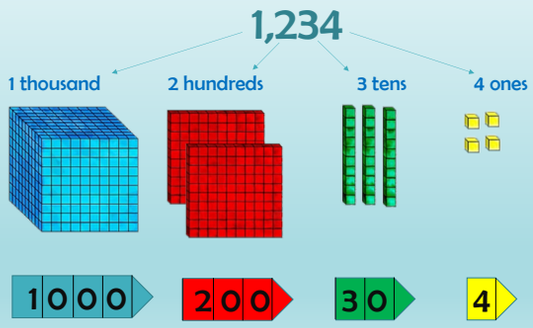
Place Value

Place value is the value of each digit in a number. It means understanding that 582 is made up of 500, 80 and 2, rather than 5, 8 and 2.



Place Value

Moving left, each column is 10 x bigger than the one before ←

→ Moving right, each column is 10 x smaller than the one before

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
M	HTh	TTh	Th	H	T	O	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
3	2	5	8	4	0	2	•	4	2	5

Three million, two hundred and fifty-eight thousand, four hundred and two **point** four two five

Ordering and comparing numbers

< >

When one value has a smaller or bigger value than another, we use a 'less than' or 'greater than' sign.

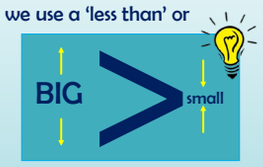
BIG > small

small < BIG

3,500 > 350

1,203 < 1,230

1.9 > 1.10



When comparing two or more numbers, you should first count the number of digits in each number.

Whole numbers with more digits in them are greater than numbers with fewer digits.

If numbers have the same number of digits, look at the individual value of the digits in the each number.

1,203 < 1,230

1000 = 1000

200 = 200

0 < 30

Ordering and comparing numbers

=

When one value has the same value as another, we use an 'equals' sign

12 + 10 = 20 + 2

15 x 4 = 60

34 – 15 = 10 + 5 + 4

12.5 = 100 ÷ 8

= means 'equivalent to'

Rounding

Up 1

9 8 7 6 5 4 3 2 1

Down to 0

Rules for rounding:

Step 1: Find the rounding place. Circle it.

Step 2: Move to the right and underline.

Step 3: 4 or less? Let it rest. 5 or more? Round up for sure!

Step 4: Change all the numbers to the right of the circle to zero.

E.g. Round 2,521 to the nearest hundred.

2,(5)3 2

2,5 3 2

4 or less? Let it rest!

2,(5)0 0

E.g. Round 9.68 to the nearest whole number.

(9).6 8

9.6 8

5 or more? Round up for sure!

1(0)

Estimating

Estimating means roughly calculating or judging a number or value.
We use our rounding skills to do this.

204 + 99 → 200 + 100 = 300

1,034 – 87.3 → 1,000 – 100 = 900

29 x 51 → 30 x 50 = 1,500

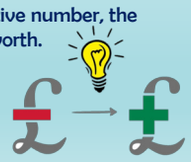
72 x 2.8 → 70 x 3 = 210

Negative numbers (counting)

-5 is less than -2

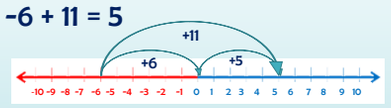
If I borrow money, I am in debt (I have a negative amount of money). The more money I borrow, the less money I have of my own! So the bigger my negative number, the smaller it's worth.

If I borrow £5, I have -5 pounds in my account.
If I borrow no money, I will have £0.
0 > -5

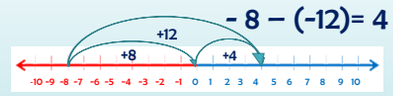


Negative numbers (calculating)

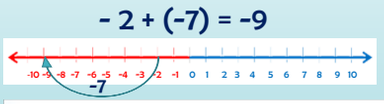
-6 + 11 = 5



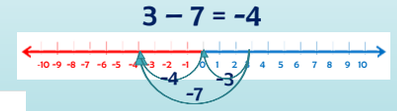
- 8 – (-12)= 4



- 2 + (-7) = -9

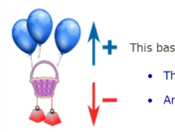


3 – 7 = -4



Balloons and Weights

Let us think about numbers as balloons (positive) and weights (negative):



This basket has balloons and weights tied to it:

- The balloons pull up (**positive**)
- And the weights drag down (**negative**)

Add positives – number increases
Subtract positives – number decreases
Add negatives – number decreases
Subtract negatives – number increases

Roman Numerals

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

Numbers are represented by putting the symbols into various combinations in different orders. The symbols are then added together, for example, I + I + I, written as III, is 3.
To write 11, we add X (10) and I (1) and write it as XI.

More than three identical symbols never appear in a row. Instead, a system of subtraction is used: when a smaller number appears in front of a larger one, that needs to be subtracted, so IV is 4 (5 - 1) and IX is 9 (10 - 1).

Modern numbers	Roman numerals	Modern numbers	Roman numerals
1	I	11	XI
2	II	12	XII and so on...
3	III	20	XX
4	IV	21	XXI and so on...
5	V	30	XXX
6	VI	31	XXXI and so on...
7	VII	40	XL
8	VIII	50	L
9	IX	60	LX
10	X	100	C

Factors, Multiples and Primes

A factor is one of two of more numbers that divides a given number without a remainder.
A multiple is a number that can be divided by another without a remainder.

5 X 4 = 20

multiple of 4

multiple of 5

factor of 20

factor of 20

The factors of 20 are:

A prime number is a whole number which can only be divided by itself and 1.
It must have exactly two factors.

2 is the only even prime!

The prime numbers up to 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97

Common factors and Common multiples

Common factors are factors that are common to more than one product.
The highest common factor (HCF) is the largest whole number which is a factor of both.

What is the highest common factor of 12 and 16?

Factors of 12 are 1, 2, 3, (4), 6 and 12.

Factors of 16 are 1, 2, (4), 8 and 16.

So the HCF is 4.

Common multiples are multiples that are common to two or more numbers.
The lowest common multiples (LCM) of two numbers is the smaller whole number which is a multiples of both..

What is the lowest common multiple of 3 and 8?

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, (24)...

Multiples of 8 are 8, 16, (24), 32, 40...

So the LCM is 24.

Square Numbers

When a number has been multiplied by itself, we say that the answer is a square number.
It's called a square number because you can show its factors as a square array.

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 = 3 \times 3 = 9$
$4^2 = 4 \times 4 = 16$
$5^2 = 5 \times 5 = 25$
$6^2 = 6 \times 6 = 36$
$7^2 = 7 \times 7 = 49$
$8^2 = 8 \times 8 = 64$
$9^2 = 9 \times 9 = 81$
$10^2 = 10 \times 10 = 100$
$11^2 = 11 \times 11 = 121$
$12^2 = 12 \times 12 = 144$

Square numbers have an odd number of factors!

Cube Numbers

If you multiply a number by itself and then by itself again, the result is a cube number.

$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 = 3 \times 3 \times 3 = 27$
$4^3 = 4 \times 4 \times 4 = 64$

$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 = 3 \times 3 \times 3 = 27$
$4^3 = 4 \times 4 \times 4 = 64$
$5^3 = 5 \times 5 \times 5 = 125$
$6^3 = 6 \times 6 \times 6 = 216$
$7^3 = 7 \times 7 \times 7 = 343$
$8^3 = 8 \times 8 \times 8 = 512$
$9^3 = 9 \times 9 \times 9 = 729$
$10^3 = 10 \times 10 \times 10 = 1000$

BIDMAS – Order of Operations

When you complete a mathematical number sentence involving several different operations then BIDMAS helps you to know which order to complete them in.

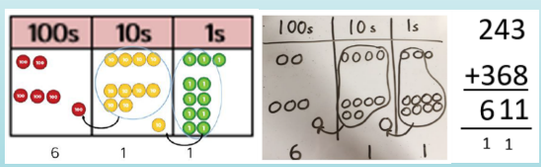
B	Brackets ( )	So... Complete anything in brackets first.	$(7 + 5)^2 + 6 \times 3$
I	Indices ^{2} ^{3}	Then do any indices/powers.	$(12)^2 + 6 \times 3$
D	Division ÷	Next do any division or multiplication. (If the calculation involves both then complete them as they appear from left to right).	$144 + 6 \times 3$
M	Multiplication x		
A	Addition +	Finally, do any addition or subtraction. (Again, if the calculation involves both then complete them as they appear from left to right).	$144 + 18$
S	Subtraction -		162

Written methods - Addition

If you cannot mentally add numbers, you may need to use the written method of column addition.

Always line up your columns carefully!

123.54 + 12.69 = 136.23



123.54
+ 12.69
136.23
1 1

When there are 10 ones in the 1s columns, we exchange for 1 ten. When there are 10 tens in the 10s column, we exchange for 1 hundred.

Don't forget to include the decimal point if your answer requires it!

Written methods - Subtraction

If you cannot mentally subtract numbers, you may need to use the written method of column subtraction.

345 – 17.6 =

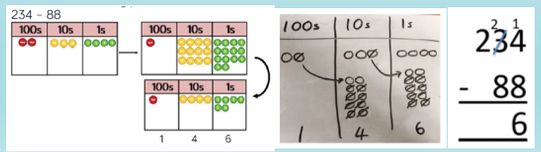
Always line up your columns carefully!

3 4 1
345.0
-017.6
327.4

Be careful when you are subtracting a decimal from a whole number.

Placing a 0 in the tenths column reminds you that the calculation is 0 – 6 which will require you to exchange from the next column.

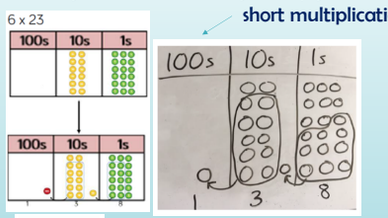
0 – 6 is not 6



Always start at the column with the least significant digit (the digit in the column with the smallest value).

Written methods - Multiplication

If you cannot mentally multiply numbers, you may need to use the written method of short multiplication or long multiplication.



6 x 23 =

23 × 6 = 138

1 1

1. Times the 6 by the digit in the ones column (6 x 3 = 18).
2. Times the 6 by the digit in the tens column (6 x 2 = 12)

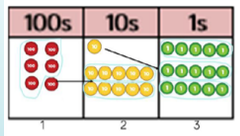
Don't forget to add any numbers that you carried into the next column!

145 x 23

```
  1 4 5
x   2 3
-------
  4 3 5
2, 9 0 0
-------
3, 3 3 5
-------
  1
```

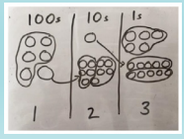
Don't forget to put the zero as a place holder!

Written methods - Division



If you cannot mentally divide numbers, you may need to use the written method of short division or long division.

1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?



$$5\overline{)615} = 123$$

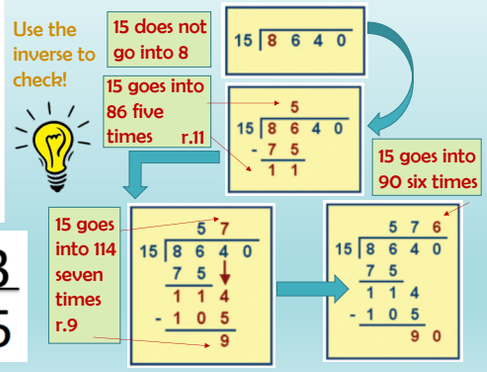
Use the inverse to check!

15 does not go into 8

15 goes into 86 five times r.11

15 goes into 90 six times

15 goes into 114 seven times r.9



Times tables

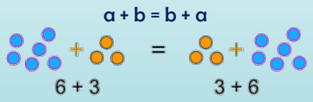
	1	2	3	4	5	6	7	8	9	10	11	12
1	**1**	2	3	4	5	6	7	8	9	10	11	12
2	2	**4**	6	8	10	12	14	16	18	20	22	24
3	3	6	**9**	12	15	18	21	24	27	30	33	36
4	4	8	12	**16**	20	24	28	32	36	40	44	48
5	5	10	15	20	**25**	30	35	40	45	50	55	60
6	6	12	18	24	30	**36**	42	48	54	60	66	72
7	7	14	21	28	35	42	**49**	56	63	70	77	84
8	8	16	24	32	40	48	56	**64**	72	80	88	96
9	9	18	27	36	45	54	63	72	**81**	90	99	108
10	10	20	30	40	50	60	70	80	90	**100**	110	120
11	11	22	33	44	55	66	77	88	99	110	**121**	132
12	12	24	36	48	60	72	84	96	108	120	132	**144**

Commutative and Associative laws

The 'Commutative laws' say that that we can change the order of the numbers in a calculation and still get the same answer...

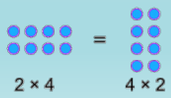
...when we add:

a + b = b + a



...or when we multiply:

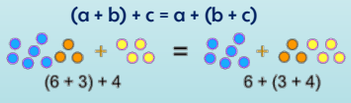
a x b = b x a



The 'Associative laws' say that it doesn't matter how we group the numbers (i.e. which we calculate first)...

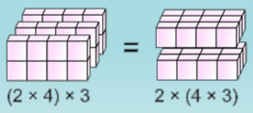
...when we add:

(a + b) + c = a + (b + c)



...or when we multiply:

(a x b) x c = a x (b x c)



Fractions

Numerator
The top digit represents the part of the whole

$\frac{1}{4}$

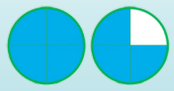
Denominator
The bottom digit represents the whole

A mixed number is made up of a whole number and a fraction.

Mixed to Improper

- Multiply the whole number by the denominator
- Add that to the numerator
- Write the result as the numerator

An improper fraction is one where the numerator is bigger than the denominator.

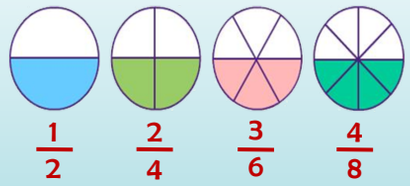


Improper to Mixed

- Divide the numerator by the denominator
- Write down the whole number answer
- Write down any remainder above the denominator

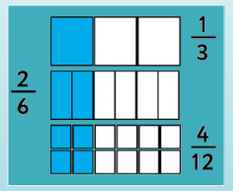
The denominator doesn't change!

Equivalent fractions and Simplifying fractions



When you x or ÷ both the top and bottom by the same number, the fractions keeps it's value.

To compare fractions, denominators must be the same!



Simplifying (or reducing) fractions means to make the fraction as simple as possible. Divide both the numerator and denominator by a common factor until you can't go any further.

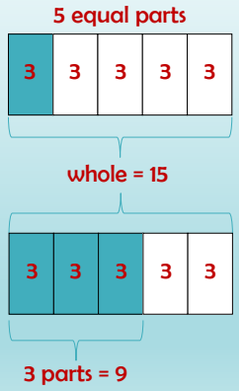
Fractions of amounts

$\frac{1}{5}$ of 15 = 15 ÷ 5 = 3

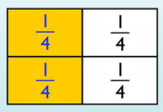
$\frac{3}{5}$ of 15 = (15 ÷ 5) x 3 = 9

÷ by the denominator then x by the numerator

5 equal parts



Adding and Subtracting Fractions



$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

The same methods work both adding and subtracting fractions.
The denominators must be the same!

$\frac{1}{2} + \frac{1}{3} = ?$

$\frac{1}{2} \times\frac{3}{3} = \frac{3}{6}$ $\frac{1}{3} \times\frac{2}{2} = \frac{2}{6}$

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

If the denominators are different...

1. Find the lowest common denominator (the lowest common multiple) for both fractions.
2. Re-write the fractions as the equivalent fractions and then calculate.

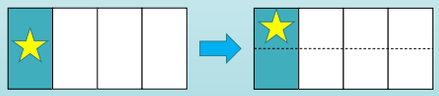
Multiplying Fractions

$6 \times \frac{1}{2} = 3$

Multiplying by a fraction is the same as finding the fraction of the amount (6 x ½ = ½ of 6)

$\frac{1}{2}$ of 6 = 3

$\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$



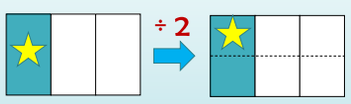
$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Method:
x numerators together
x denominators together

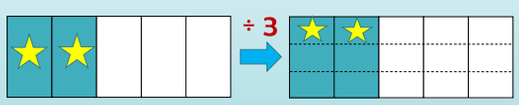
$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

Dividing Fractions

$\frac{1}{3} \div 2 = \frac{1}{6}$

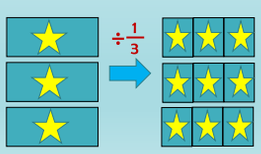


$\frac{2}{5} \div 3 = \frac{2}{15}$



$3 \div \frac{1}{3} = 9$

How many thirds fit into 3?
Answer = 9



Use the bar model to draw your whole number or fraction. Then divide it into parts.

Fractions, Decimals and Percentages

Fraction	Decimal	Percentage
1	1	100%
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	0.33	33.3%
$\frac{1}{4}$	0.25	25%
$\frac{1}{5}$	0.2	20%
$\frac{1}{10}$	0.1	10%
$\frac{3}{4}$	0.75	75%
$\frac{3}{5}$	0.6	60%

Fractions → Decimals
Numerator ÷ Denominator
$\frac{1}{2}$ → 1 ÷ 2 = 0.5

Decimal → Percentage
x 100
0.5 x 100 = 50%

Percentage → Decimals
÷ 100
50% ÷ 100 = 0.5

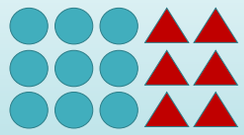
Percentage → Fraction
$50\% = \frac{50}{100} = \frac{1}{2}$

Turn the % into a fraction by putting it over 100. Then simplify.

Percent means 'parts per 100'

Ratio

A ratio compares values, telling us how much of one thing there is compared to another thing.

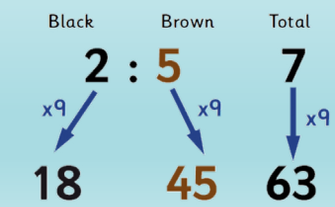


The ratio of ● to ▲ is 9 : 6

This can be simplified to 3 : 2

If the total increases by 9, then so must each of the parts.

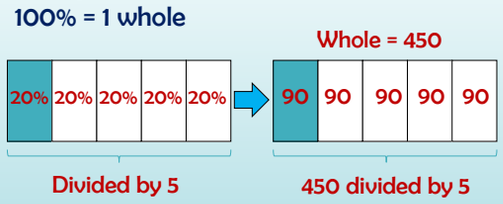
In ratio, amounts either get x bigger (multiplied) or x smaller (divided). All parts must be increased or decreased by the same amount.

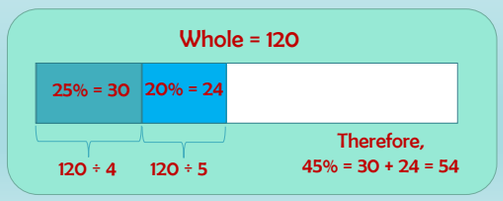


Percentages of amounts

Percentage	Method
100%	÷1
50%	÷2
25%	÷4
20%	÷5
10%	÷10
5%	÷20 or ÷10 then ÷2
1%	÷100

100% = 1 whole





Algebra

Algebra is where letters and symbols are used to represent numbers and quantities in equations (number sentences).

3a + 12 = 21

a	a	a	12
21			

3a = 9 (-12 from both sides)

a	a	a	12
9			-12

a = 3 (÷both sides by 3)

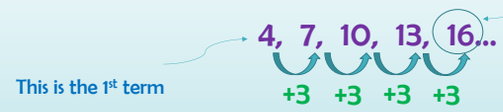
a	a	a
3	3	3

Sometimes we are given the value of the letter of symbol and asked to solve the equation.
Other times we are asked to find the missing value of the letter or symbol.

In algebra, you must keep the equation balanced.
So if you divide one side by 5, you must do the same to the other!

Linear Sequences

Each number in the sequence is called a term



This is called a linear sequence because the difference between each term is always the same (this is known as the common difference).

Each linear sequence has a n^{th} term.
For this sequence:

The n^{th} term could be any number in the linear sequence.

n^{th} term = 3n + 1
(1 x 3) + 1 = 4
(2 x 3) + 1 = 7

The 10^{th} term in this sequence will be
(10 x 3) + 1 = 31

How many of these do you remember? Test yourself

Date	I need to learn		