

**Properties of Ultrasonic Waves**

Phil Picton

School of Science and Technology

**Division of Engineering**

**University of Northampton**

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## Properties of an ultrasonic beam

You may have studied Ultrasonic Testing, have a PCN Level II or III in Ultrasonic Testing, or a qualification and experience from elsewhere. Whatever is the case, the subject of Ultrasonic Testing shouldn’t be new to you.

You will have come across some of the equations that are used in ultrasonic testing, but have probably not seen any derivation. The following sections give a more detailed derivation of the equations that are used to describe the ultrasonic beam.

The material towards the end introduces the concept of software modelling to students who have not seen this software before. If you did software modelling before, the material will be useful revision for you.

## Spherical sound waves in an elastic solid

The equations that are used in ultrasonic testing have been derived within the field of Acoustics or Physical Acoustics. There are various models of sound waves travelling through different media with various types of source. The specific case that is relevant to ultrasonic testing is usually taken to be “a circular piston with infinite baffle”. We assume that the ultrasonic probe is circular, and that it is in perfect contact with the material. The probe is regarded as the “piston” which forces the material to vibrate. The way the piston is designed and attached to the material means that the sound or vibrations mainly travel into the material. Theoretically this is like a piston with an “infinite baffle”. The “baffle” prevents the sound travelling in any other direction other than forward into the material. The waves produced by the probe are spherical waves, but again because of the way the probe is connected to the material, the waves only travel forward so you only get half a sphere (a hemisphere) projecting into the material.



probe

baffle

hemispherical waves

Figure 1 Circular piston with infinite baffle producing hemispherical waves

In order to derive some of the equations that describe these waves, we need to define some terms. Firstly, we need to define the quantity Q as the strength of the wave. It is defined as the rate that the volume of material is moving and is usually calculated as the area times the velocity. If we have a specific area, 1 mm2 for example, then the amount of the wave that passes through that area in a fixed time, 1 ms, is a measure of the waves strength. A stronger wave would push more material through, so we could either say that it pushes the same volume through only quicker, or it pushes a bigger volume through (so a bigger area).

The area of the surface of the sphere with a radius of “r” is 4πr2, so that Q = 4πr2u(r), where u(r) is the velocity of the material at a distance “r” from the centre of the probe. For a hemisphere the area would be half of this so the strength is Q = 2πr2u(r).

**Problem 1**

What are the units of strength, Q?

Next we need to know what the pressure would be at any point in the material. Rather than calculating the pressure caused by the whole probe in one step, we start off by considering the pressure caused by a single infinitesimally small point. We do this by imagining a spherical source that is very small. I won’t go through all of the maths, but it can be shown that the pressure at a distance “r” from this small point would be:

So p(r) is the pressure at any point (in Pascals or N/m2) at a distance r (in metres) from the source;

ρ is the density of the material (in kg/m3);

Q is the strength of the source (in m3/s);

k is the wave number which equals ω/c (in m-1);

c is the velocity of the wave front (in m/s);

ω is the angular frequency of the wave and equals 2πf (in rad/s);

t is time (in seconds).

NOTE: The wave number, k, is a quantity used to describe waves. It indicates how many wavelengths there are in a fixed distance. In some cases the wave number is defined as k = f/c = 1/ λ and indicates how many wavelengths you get in one meter. In the derivation in this Learning Package we are using the more conventional definition in which k = ω/c = 2π/λ, and therefore represents the number of wavelengths in 2π meters. The units of the wave number are m-1 or cm-1 as they indicate how many wavelengths per unit length.

**Problem 2**

For an ultrasonic wave with a frequency of 6 MHz travelling through steel with a velocity of 5960 m/s, what is the wave number?

For the rest of the equation I need to quickly explain complex numbers where “j” is used.

Mathematicians wondered for a long time what the square root of a negative number could be. A positive number is relatively easy. The square root of 9 is either +3 or -3 since +3 x +3 = 9 and -3 x -3 = 9. So can -9 have a square root?

To solve this mathematicians invented an imaginary number and called it “i”. This term “i” stood for the square root of -1. This meant that i x i = -1. In Engineering, and particularly in electronics, the symbol “i” usually means current, so instead the symbol “j” is used to mean the square root of -1. All very interesting, but what has that got to do with pressure in a solid material?

We use the j term when we want to do maths with quantities that have two values that we want to keep separate. In complex numbers we have numbers that have a real and imaginary part. For example the complex number:

X = 3+ j4

has a real part which has a value of 3, and an imaginary part which has a value of 4. We usually represent these numbers on a graph, with the real part along the horizontal axis and the imaginary part along the vertical axis, as shown in Figure 2.

The real and imaginary parts are kept separate – they cannot be combined. We can perform arithmetic on complex numbers, and the results will also be complex.

For the issue of pressure waves, we have assumed a 2-dimensional problem where the wave is propagating through the material. If we have a probe at the top, the wave propagates down through the material. However, it doesn’t just go vertically down, but also spreads out horizontally. So the direction of the oscillations will be at an angle to the probe. This is captured in the equation by using complex numbers which can then be used to model a 2-dimensional wave.

A second use of the complex variable “j” is that it can be used to calculate the phase relationship between waves. If the wave at the probe is assumed to be the original wave, then all other waves in the material can be described with a phase shift relative to this original wave. The “j” at the start of the equation does just this. It indicates that there is a phase shift of 90 degrees for all waves compared to the original. Further phase is added by the other terms.

1

2

3

4

-1

-2

-3

-4

-1

-2

-3

-4

1

2

3

4

0

Real

Imaginary

X=3+j4

Figure 2 Graph of complex numbers

A complex number can be written as a + jb, but can also be written in polar coordinates as a length, R, and an angle, θ. The length would be and the angle as θ =Tan-1(b/a). For the example X = 3 +j4, the value of R would be:

θ = Tan-1(b/a) = Tan-1(4/3) = 53.13o

This is illustrated in Figure 3.

If we start with R and θ then we can calculate a and b as:

a = Rcos(θ) and b = Rsin(θ).

So another way of writing a complex number is:

X = Rcos(θ) + jRsin(θ)

Finally, you will have come across exponentials such as e-x, which is the inverse function of the natural or Napierian logarithm. The new twist here is that the index on the exponential can be complex. We can write the complex number above as:

X = Re-jθ

1

2

3

4

-1

-2

-3

-4

-1

-2

-3

-4

1

2

3

4

0

Real

Imaginary

X=3+j4

R

θ

Figure 3 Polar representation of a complex number

**Problem 3**

a) What is the complex number X = -4 + j7 in polar form?

b) What is the complex number X = 8e-j42 in the form X = a + jb?

In the equation for the pressure we have two exponential terms. The first is:

e-jkr = cos(-kr) + jsin(-kr)

This introduces a phase shift depending on the distance from the source, r, and the frequency of the wave, expressed in terms of the wave number, k. The second term is:

ejωt = cos(ωt) +jsin(ωt)

This shows how the pressure changes with time, and is essentially a sine wave with a relative phase shift compared to the wave at the probe.

To complete the derivation we also need an expression for u(r), the velocity at a distance r from the source. From physics we can use the following relationship:

What this shows is that how the velocity changes with time (the acceleration, ) depends on the density, of the material and the change in pressure over distance, . As an analogy, imagine a spring. As an object moving with a velocity hits the spring, the force (or pressure) causes the spring to compress. Meanwhile, the object is being slowed down by the spring, so its acceleration is negative. As the spring gets more compressed the greater is the opposing force.

Using this equation, and the equation for pressure, we can derive the equation for the velocity at a point, which is:

So, this gives us the pressure and velocity from a point source. What we actually have is a piston, which we could think of as lots of points over a circular surface. To find the resultant we take the pressure from a small area on the piston, and integrate it over the whole surface using calculus.

Whole area, S

Small area, dS

Distance, r

Pressure, dp(r)

Piston

Figure 4 Pressure, dp(r), at a distance r caused by the small area, dS

The pressure at a distance r from the small area is called dp(r). Recall that p(r) is the pressure at a distance r from a single point. To indicate that we are looking at a small contribution to the pressure from the small area we indicate the small pressure as dp(r) where the “d” means a small part. In general if we have variables x and y where y is some function of x, we can show a small change in y due to a small change in x as dy and dx.

In the equation for pressure we show the strength as Q. We said that Q is the volume strength, and is found by multiplying the velocity at a point by the area. The area of the whole probe or piston is S, so a small area would be dS. The velocity at a distance r is u(r). So the strength is u(r)dS. The equation for the pressure due to the small area is then:

To find the pressure at a point a distance r from the source, we integrate over the whole area. It turns out that this is actually too hard, even for advanced mathematicians. So, what we look at instead are two special cases.

### Pressure along central axis

The first solution is the pressure along the central axis in front of the probe. The solution is the equation used by Krautkramer and Krautkramer (1990) in their book. In this equation D is the diameter of the probe and x is the distance along the central axis.



This equation gives a fairly good description of the pressure amplitudes in the near and far zones. We can derive the equation for the near zone using this equation. It has a maximum value when the value within the sine function is equal to 90 degrees, or π/2 radians. So:

With a bit of rearranging, this becomes:

Often the diameter is much bigger than the wavelength in which case the equation is simplified to:

This is the equation that is normally used for the length of the near zone.

**Problem 4**

A circular probe with a diameter of 1.4 cm and a frequency of 2 MHz is used in a test on Aluminium using compression waves. What is the predicted value of the distance to the edge of the near zone using both of the above equation?

### Directivity function

The second equation for the pressure that can be solved assumes that the distance r is quite large, so that the pressure is in the far zone. The solution is then:

In this equation U0 is the velocity of the wave at the probe, *θ* is the angle relative to the central axis below the probe, and J1[x] is what is known as the first Bessel function, and the whole expression within the square brackets is known as the directivity term. The new variable, a, is the radius of the probe.

Of interest is the fact that the directivity term has maxima and minima, and is both positive and negative, which means that at certain values it equals zero. This is shown in Figure 5.

Figure 5 Directivity term

The first time that the directivity term equals zero occurs when *ka*sin(*θ*) = 3.83. Rearranging, we get:

sin(θ) = 3.83/ka

k is the wave number which equals ω/c (m-1);

c is the velocity of the wave front (m/s);

ω is the angular frequency of the wave (rad/s) and equals 2πf;

a is the radius of the probe (in metres) and equals D/2, where D is the diameter of the probe (in metres);

So k = 2πf/c, and 3.83/ka = 3.83c/2πfa = 0.61c/fa = 1.22λ/D

sin(θ) = 1.22λ/D

The angle, θ, is the angle relative to the centre of the probe, so we would normally think of this as the half-angle. The equation in this form is the same as the equation that is often found for the value of the half angle i.e. the width of the ultrasonic beam. The constant, 1.22, is often given the latter k, which is unfortunate as that’s what we’ve used for the wave number. To distinguish it, I will call it kθ.

The following table shows specific values of the directivity term and x, together with the computed value of constant, kθ.

|  |  |  |  |
| --- | --- | --- | --- |
| x=kasin(θ) | 2J1(x)/x | dB | kθ |
| 0 | 1 | 0 | 0 |
| 1.75 | 0.66 | -3.6 | 0.56 |
| 2.2 | 0.5 | -6 | 0.7 |
| 3.39 | 0.1 | -20 | 1.08 |
| 3.83 | 0 | -∞ | 1.22 |

The first row shows the value of the directivity function when the value of x is zero, corresponding to θ = 0. The directivity function is at its maximum value here. The last term, when kθ = 1.22, we have seen already. This corresponds to the edge beam.

The value in the table for kθ = 1.08, corresponds to the directivity function having a value of 0.1 or one tenth of the peak value. In decibels this would be 20 dB down.

If we look at a typical source of information on ultrasonic testing, such as the Handbook of Nondestructive Evaluation, by Hillier (2001) we find:

“The above equation includes the constant 1.22. This calculates the beam spread to the absolute limit of the beam where sound ceases to exist. This is not a practical limit for the ultrasonic practitioner because if sound doesn’t exist it can’t be detected or measured. In practice, it is more usual to replace the constant 1.22 with either 0.56 or 1.08. The 0.56 value predicts the limits of the beam where the sound has dropped to one half of the intensity at the beam centre. The 1.08 value defines the limits where the sound is one tenth of that at the beam centre.”

Our model agrees with this statement for the terms when kθ = 1.22 and 1.08. However, the model predicts that with kθ = 0.56 the pressure will be two thirds of the peak value not a half. If we want to detect the beam when it has dropped to half its peak value the model predicts that we should use kθ = 0.7. Many sources on ultrasonic testing, such as the one above, use 0.56, but this would appear to be incorrect.

The figures below are taken from the Internet, <http://www.ndt.net/ndtaz/files/ut_formula/ut_formula.php>

-1.5dB k=0.37  
-3dB k=0.51  
-6dB k=0.7  
-10dB k=0.87  
-12dB k=0.93

**Problem 5**

A circular probe with a diameter of 1.4 cm and a frequency of 2 MHz is used in a test on Aluminium using compression waves. What is the predicted value of the half angle at which the beam has dropped in magnitude by 6 dB?

If we use the directivity function to plot all the points in the material below the probe where the pressure drops to zero, then we end up with a plot like Figure 6. This shows the main beam directly below the probe, and some side-lobes, where signals may be received, but they are small in comparison to the main beam.

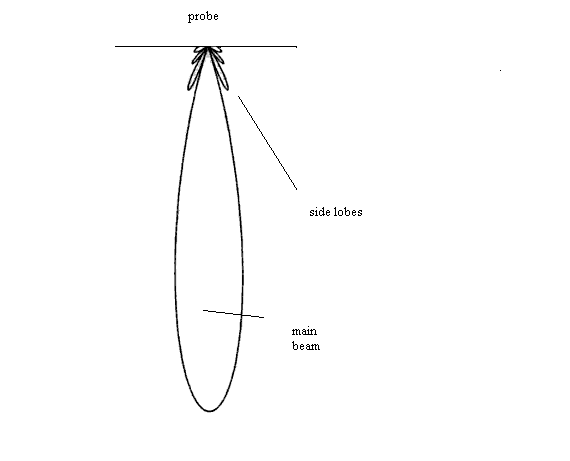


Figure 6 Main beam and side lobes

## Modelling the beam

In the previous sections the simplest model, although it probably doesn’t look simple, was found. The equation shows the value of the amplitude of the pressure along the central axis of the wave at any point.



In this equation D is the diameter of the probe, and x is the distance along the central axis from the probe.

Figure 7 Plot of the pressure amplitude along the central axis of the beam

Figure 7 shows a plot of the equation where the y-axis is pressure amplitude, p, and the x-axis is distance from the probe, x. It has the characteristic shape that is often shown of the amplitude of the wave along the beam axis. The peak appears at the point which defines the end of the near zone (at a distance of 9 mm) and the start of the far zone.

In ultrasonic testing, we are usually interested in the detection of flaws in the far zone, and tend to avoid looking for flaws in the near zone because of the difficulties in detecting accurately. The near zone is defined as:



For the graph above, D = 6 mm and λ = 1 mm. Therefore the far zone is defined as being x >> N, i.e. the distance from the probe is much greater than the length of the near zone. Under these conditions the equation for the pressure can be simplified.

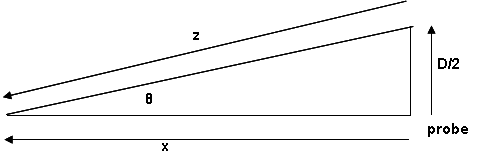


Figure 8 Distance x from the probe of diameter D

Figure 8 shows a right-angled triangle. The vertical length is half the diameter of the probe, D/2, and the horizontal length is the distance from the probe, x. The hypotenuse length, z, is given by:



For large distances from the probe, the value of x is going to be almost the same as the value of z. Therefore we can say that:



Now substitute for z2 using the equation above:



The original equation for the pressure was:



Which includes the term:



This term equals (z - x), therefore:



**Problem 6**

With D = 6mm, calculate the difference between the two terms for (z – x) in the above equation for values of x of 9, 18, 27 and 36mm. Are they approximately equal?

The next step is to say that at large values of x, i.e. at large distances from the probe, we can make the following simplification:



This function is plotted in Figure 9. It shows that for distances in the far zone, the approximation is reasonable.

Figure 9 Approximation of the pressure in the far zone

The function is always positive at these distances, so we can get rid of the absolute value brackets, and note that we could also substitute the equation for the length of the near zone, N.



This shows that the pressure falls off in inverse proportion to the distance measured in multiples of N.

So far, the assumptions that have been made are that the probe is circular, with diameter D, and that in the far zone the distance from the probe is large compared to the diameter. Since the probe is circular, its area, S, is:



The pressure is now expressed in terms of the area of the probe, the wavelength of the ultrasonic wave and the distance from the probe.

**Problem 7**

For the probe described so far, where D = 6mm and λ = 1mm, and using the equations derived so far, what is the pressure at a distance of x = 36mm assuming that p0 = 1?

### Small disc-shaped flaw

When the ultrasonic wave hits the flaw, the wave is reflected. As the flaw is small relative to the width of the beam, much of the beam continues until it hits the back wall. However, the beam that is reflected from the flaw is like a new beam being transmitted from a new circular source. This means it can be modelled just as the original beam from the probe, only now it’s the area of the flaw that is used. The value of p0, the initial strength of the beam, equals the value of p when the beam arrives at the flaw from the probe. If we use the subscript s for the original source, and the subscript f for the flaw, then the equation for the pressure at any point in the far zone, x, is:



If the flaw is at a distance x from the probe, then this equation represents the pressure of the wave as it reaches the flaw. The reflected beam will have the same pressure, so this also represents the initial pressure of the beam that is reflected from the flaw. At the receiver, which is a distance x away from the flaw, the received beam will have a pressure of:



In other words, the pressure of the wave as it arrives at the probe, having been reflected by the small disc-shaped flaw, is proportional to the area of the probe and flaw, and inversely proportional to the square of the distance of the probe to the flaw, and the square of the wavelength of the ultrasonic beam.

### Back-wall echo

If there is no flaw, and all of the beam is reflected off the back wall, the pressure is given by the equation:



This is the original equation for the pressure, with the distance made equal to 2x, since the beam has travelled from the probe to the back wall over a distance x, and back to the probe again, another distance x, giving a total of 2x.

**Problem 8**

For a probe of diameter 6mm and wavelength of 1mm, using the equations derived so far, what would the back wall echo be if the thickness of the material is 50mm and p0 = 1?

### Distance-Gain-Size (DGS) diagrams

Krautkramer and Krautkramer (1990) used the equations that we have just derived to construct his distance-gain-size diagrams.

Converting areas back to diameters, where:





If we make:



Then:





This gives the pressure from the reflected beam in terms of the distance from the probe in multiples of N, that’s the value of H, and the diameter of the flaw in terms of multiples of the diameter of the probe, which is the value of G.

For the back wall echo, the same is applied.



The DGS diagram is constructed using these equations for the small disc-shaped reflector, and the back wall echo.

## Attenuation

The equations derived so far, take into account the fact that the pressure on the material along the beam falls off inversely proportional to the distance travelled. However, there are other effects which have not been taken into account, and these are usually summarised as the attenuation. Essentially, the pressure also falls off exponentially as the distance travelled. So:



Where alpha is the attenuation coefficient measured in Nepers/mm. If we combine the two equations for the pressure we get:

For small disc-shaped reflectors:



Note the fact that the distance travelled in the exponential term is 2x as the beam has travelled to the reflector and back again.

For the back wall:



These equations that are referred to as the Ermolov equations.

**Problem 9**

Using the Ermolov equation, calculate the size of the back wall echo for a probe of diameter 6mm, wavelength of 1mm, thickness of 50mm and attenuation constant of 0.03 Nepers/mm and p0 = 1.

## Software simulation

Ed Ginzel of the Materials Research Institute in Waterloo, Ontario, Canada produced a paper which describes the Ermolov equations (Ginzel, et al., 2002). As well as the equations for the disc-shaped reflector and the back wall, the paper includes equations for other shaped targets.

The paper also includes a link to free downloadable software at:

<http://www.ndt.net/article/v07n01/ginzel/ermolovsolutions_v1-1.zip>

If you go to this website, you can download this software yourself, and use it.

You can download the zip file form this website, which can then be installed by clicking on “SETUP”.

When you run the program, click on the opening screen.

By entering the parameters of a particular material, such as the acoustic velocity and the attenuation coefficient, you can then enter data about a probe, such as its diameter and frequency.

For the moment, don’t change anything, just use the values that are already there.

Under the top “Select Target Type”, choose “flat defect”, and then put in a diameter of 2mm, and the distance along the beam as 500mm. Then click on “Update”.

It’s worth noting that whenever you enter or change any of the data, you have to click on “Update” before anything changes.

Next, chose the next “select target type” down, and choose “infinite plane” and set the distance to 600mm. Don’t forget to click on “Update”.

So, what we’ve done is set up the system so that the back wall is 600mm away from the probe, and we’ve put a small flat disc 500mm away from the probe. The screen shows that the echo from the back wall would be 112.4 dB down compared to the initial pulse, and that the echo from the flaw would be 124.3 dB down. In other words, if we set the main bang to be 80% FSH, then we would have to turn up the gain by 112.4 dB to get the back wall echo up to the 80% mark, and turn the gain up by 124.3 dB to gat the echo from the flaw up to the 80% mark.

Next, click on “Target Amplitude Plots”. This shows the complete picture of the amplitude of the echo as a function of distance.

The y-axis is in dB, but can be changed to linear if necessary by clicking on “voltage drop”. The x-axis is measured in multiples of the Near Field distance, N. You can see that a change occurs at N = 1. Beyond that, the curves follow a smooth decline. Prior to that, in the near zone, the lines are fuzzy. This is a band that shows the limits of the maximum and minimum values that can be expected in the Near Zone.

One minor point that has to be mentioned. The attenuation coefficient that you enter in the software is shown as having units of dB/mm. This is an error, and it should say Nepers/mm.

**Problem 10**

Repeat Problem 9 using the software. Assume that the probe has a frequency of 6 MHz, that the velocity is 5960m/s.

## References

Krautkramer, J. & Krautkramer, H. (1990). *Ultrasonic Testing of Materials*. Springer-Verlag.

Hillier, C.J. (2001). *The Handbook of Nondestructive Evaluation*. McGrawHill

Ginzel, E., Ginzel, R. & Kanters, W. (2002). Ermolov Sizing Equations Revisited. *NDT.net*, *7*(1). <http://www.ndt.net/article/v07n01/ginzel/ginzel.htm>

## Answers to the Self-Test Questions

**Problem 1**

Strength Q is defined as the area times the velocity. Area would be measured in m2 and velocity in m/s, so the strength would be m3/s.

**Problem 2**

Wave number k = ω/c = 2πf/c = 2 x 3.142 x 6 x 106/5960 = 6325.35 m-1

**Problem 3**

1. What is the complex number X = -4 + j7 in polar form?

θ = Tan-1(7/(-4)) = Tan-1(-1.75) = -60.26o

This value for the angle is incorrect. If we look at the diagram in Figure 1, for example, a complex number with a real value of -4 and an imaginary value of +7 would be in the top left part of the graph. The angle should therefore be 180 – 60.26 = 119.74o. The reason for this error is because my calculator cannot tell the difference between the situation where the real part is negative and the imaginary part is positive, and where the real part is positive and the imaginary part is negative – for example X = 4 – j7. Both give the value of -1.75, so that when the angle is calculated it is for the second example not the first. By reference to the graph you should be able to work out what the correct angle should be.

1. What is the complex number X = 8e-j42 in the form X = a + jb?

a = Rcos(θ) = 8cos(42) = 8 x 0.743 = 5.95

b = Rsin(θ) = 8sin(42) = 8 x 0.669 = 5.35.

X = 5.95 + j5.35

**Problem 4**

The wavelength = velocity/frequency = 6374/2000000 = 0.0031875 m or 3.1875 mm

**Problem 5**

As we saw in Problem 4.4, the wavelength is 3.1875 mm. The diameter is 14 mm. Using:

sin(θ) = 0.7λ/D = 0.7 x 3.1875/14 = 0.159

θ = 9.17o

**Problem 6**

Substituting D = 6:



|  |  |  |
| --- | --- | --- |
| x |  |  |
| 9 | 0.486833 | 0.5 |
| 18 | 0.248288 | 0.25 |
| 27 | 0.166155 | 0.166667 |
| 36 | 0.124784 | 0.125 |

The table shows that the two sides of the equation are approximately equal.

**Problem 7**



**Problem 8**

Substituting D = 6, λ = 1 and p0 = 1, x = 50mm:



**Problem 9**



**Problem 10**

The relative amplitude is 1.42E-02 which is the same as 0.0142.